

Comparison of efficiency of extraction of built-up areas in aerial images using fractal analysis and morphological granulometry

*Porównanie efektywności wyodrębniania terenów zabudowanych na obrazach
lotniczych przy pomocy analizy fraktalnej i granulometrii morfologicznej*

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Abstract

The paper presents a comparison of results of the automatic extraction of built-up areas, based on fractal analysis and granulometric maps, in the aerial images. Built-up areas as a land-use class can be clearly seen in an aerial or satellite image, due to its high granularity, but for the same reason they are very difficult to extract using a “traditional” non-contextual, pixel-based classification. Both approaches presented in the paper, using fractal analysis and morphological granulometry, base generally on a pixel-based classification, but performed on images previously processed using these two types of processes. Fractal analysis consists in an empirical computing of fractal dimension of parts of an image, using a box-counting method. Such an approach generates an image where pixel values are equal to a fractal dimension values of their neighbourhood. Since we can interpret a fractal dimension as a level of granularity, a simple reclassification of such an image can improve a performance of an automatic extraction of built-up area effectively. The approach based on a morphological granulometry creates a number of granulometric maps – images where pixel values mean an amount of objects of certain size in a set neighbouring fragment of an image. This way a number of these images can be processed using a pixel-based classification, to perform an effective extraction of built-up areas in an image. The results of the presented approaches have been compared to the reference mask obtained basing on a visual interpretation of the image.

Keywords: remote sensing, mathematical morphology, fractal analysis, classification, digital image processing.

Słowa kluczowe: teledetekcja, morfologia matematyczna, analiza fraktalna, klasyfikacja, cyfrowe przetwarzanie obrazów.

Artykuł przedstawia porównanie wyników automatycznego wyodrębnienia terenów zabudowanych na podstawie analizy fraktalnej oraz map granulometrycznych. Tereny zabudowane jako klasa pokrycia terenu są

łatwo wyróżnialne na zdjęciach lotniczych i satelitar-nych w procesie wizualnej interpretacji, ze względu na charakterystyczną silną teksturę obrazu. Z tego samego względu są one trudne do wyodrębnienia w procesie klasyfikacji spektralnej. Obydwa przedstawione w artykule podejścia opierają się na klasyfikacji pikselowej, jednak z wykorzystaniem obrazów utworzonych w wyniku jednej z dwóch metod analizy teksturowej. Analiza fraktalna polega na empirycznym obliczeniu wymiaru fraktalnego poszczególnych fragmentów obrazu na podstawie metody liczenia pudełkowego (ang. box-counting). Wymiar fraktalny możemy interpretować w kontekście ziarnistości (tekstury) obrazu, prosta reklasyfikacja takiego obrazu

pozwała istotnie podnieść skuteczność wyodrębnienia terenów zabudowanych na zdjęciach. Podejście oparte na granulometrii obrazowej prowadzi do utworzenia pewnej liczby map granulometrycznych – obrazów, których wartości pikseli oznaczają ilość obiektów określonych rozmiarów w otoczeniu poszczególnych pikseli. Mapy granulometryczne są następnie przetwarzane z wykorzystaniem metod klasyfikacji pikselowej, co również pozwala wyodrębnić tereny zabudowane z dużą skutecznością. Wyniki przedstawionych podejść zostały porównane z referencyjną maską terenów zabudowanych opracowaną na podstawie wizualnej interpretacji obrazów.

Introduction

Pixel-based spectral classification is a well-known method to extract different classes of land cover in aerial and satellite images (Jensen, 1996; Chmiel, 2002). So are its qualities and disadvantages. Recapitulating briefly, spectral classification is relatively (comparing to other, approaches more complex than pixel-based one) simple and undemanding, but generally, it bases only on pixel values, representing, in a manner of speaking, colours of objects. When comparing it to the objects features, which are taken into account in the photo-interpretation process, like size, shape, texture etc. it is obvious, that an important amount of information is missed. This is why spectral classification is relatively effective when dealing with general land cover classes, possible to extract basing only on their colours, but is inefficient as a tool to extract land cover or land use classes, like built-up areas or orchards, basing on features different than spectral information (e.g. size, shape or texture).

In the recent years many new classification algorithms to resolve the problem described above have been developed. One of the most popular and, also, successful is an object-based classification, basing on classification of objects – segments extracted precedently, during a segmentation process (Blaschke *et al.*, 2000, Walter, 2004). Such an approach allows to take into account also characteristics other than colour (pixel value) of image fragments, so it is an important step forward in a development of automatic (or semi-automatic) classification methods.

The aim of this paper is to present a different approach of a contextual classification and to investigate its efficiency. The approach consists in 2 steps: a processing of an original satellite image to obtain a contextual information about a granularity of pixels' neighbourhoods followed by a pixel-based classification. The main idea of this process is, that though a pixel-based classification itself is non-contextual, the whole process becomes contextual, due to a reason, that it usually deals with a contextual information.

Two different proposition of contextual processes, giving an information about granularity of the image are presented, investigated and compared: Fractal ana-

lysis, consisting in application of box-counting method, and granulometric maps – an interesting tool derived from mathematical morphology or, to be more specific, morphological granulometry.

Test area and data

In the presented research, two aerial photos (both, natural colors, scale 1:26000, scanned to spatial resolution 0.65m x 0.65m) have been used. Images present partially built-up fragments of two town placed in Mazovian region: Jabłonna and Karczew. The image presenting Jabłonna had a size of 8109 x 10913 pixels, and the image presenting Karczew had a size of 8446 x 11364 pixels. In each town lives about 16000 people. Additionally, Jabłonna, due to its localisation, in a direct neighborhood of Warsaw – capital city of Poland is exposed to a strong suburban pressure. Both test areas are shown in the figure 1.

For the further research (presented in the chapter 3), two reference binary masks of built-up area has been created during a visual interpretation of the images. Built-up area have been defined as a whole, including buildings, and the mid-spaces, such as courtyards, sidewalks, green areas, etc., so it was matched to a visual interpretation. The masks have been considered faultless, basing on a fact, that the visual interpretation of this kind of land use, taking into account contextual attributes (like texture, size and shape of the elements of the image) might be considered as sufficiently accurate. These masks are presented in the figure 2.

Method

Methodology of the research is presented generally in the figure 3. As it can be seen, both presented approaches are based generally on the same scheme.

As the figure 1 shows, the source image will be processed using fractal analysis or morphological granulometry, independently, then two classes extracted using these approaches, built-up area and non-built-up area) will be extracted in an every product of the prece-



Fig. 1. Test area, a) Jablonna, b) Karczew.

Ryc. 1. Obszary testowe, a) Jablonna, b) Karczew.

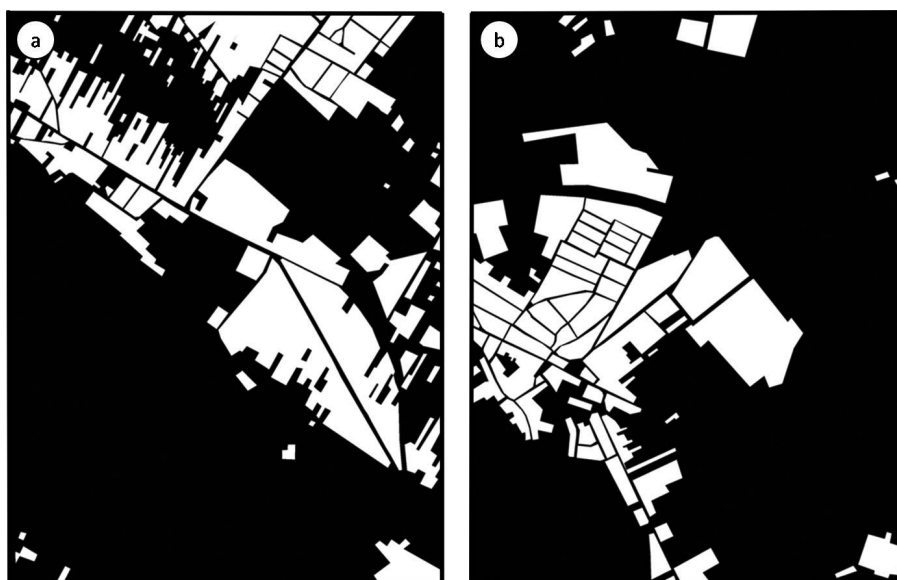


Fig. 2. Reference masks, built-up area marked with white color, a) Jablonna, b) Karczew.

Ryc. 2. Maski referencyjne, obszary zabudowane zaznaczone białym kolorem, a) Jablonna, b) Karczew.

ding processes, using ISODATA algorithm, finally, the results will be compared to the referential built-up area mask, produced in the process of a photo-interpretation. As mentioned above, built-up area masks have been matched to a visual interpretation of the image. They have been defined as a whole area, including buildings, and the spaces between them.

What differs two approaches presented above: fractal analysis and morphological granulometry, is the method

to obtain images containing a contextual information on granularity in a pixel neighbourhood. These methods are briefly presented below.

Fractal analysis. Fractal analysis might be a way to characterize an image (or objects in an image), by computing its (their) fractal dimension. Fractal dimension (or similarity dimension) is an index for characterizing fractal patterns (or complexity), by comparing how detail in a pattern changes with the scale at which it is

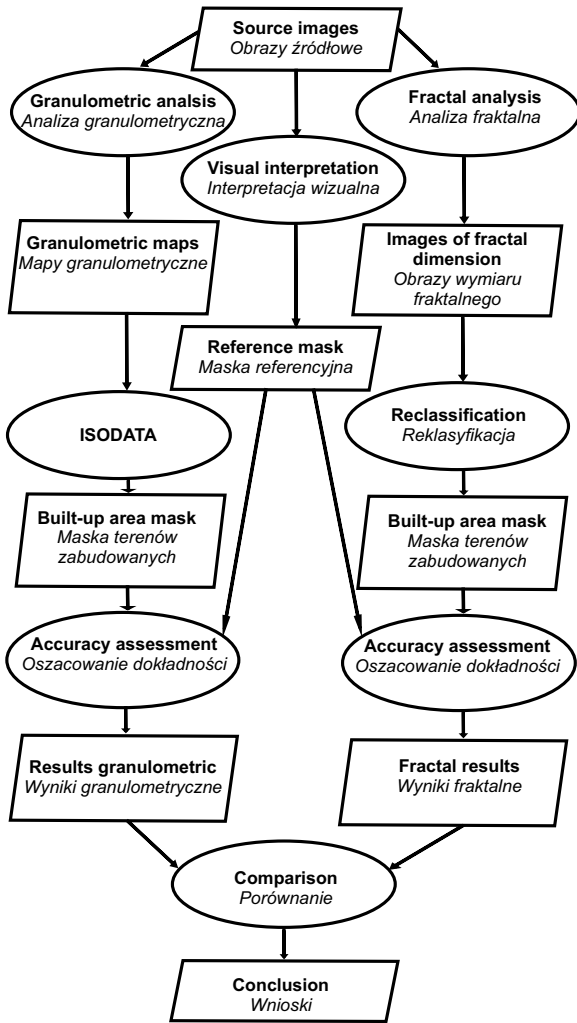


Fig. 3. Methodology of the research.

Ryc. 3. Metodyka badań.

measured (Falconer, 2003). It can be expressed using the following formula:

$$D = \frac{\log N}{\log \varepsilon} \quad (1)$$

where D is a fractal dimension, N is a size of a set in an image (number of pixels valued 1 in a binary image), and ε is a scale factor.

Propositions of applying fractal analysis for cartographic and remote sensing data processing appeared before e.g. in (Sun *et al.*, 2006; Shen, 2002; Encarnação *et al.*, 2012)

For the purpose of this paper, box-counting method is applied. It is a mean to calculate a fractal dimension empirically. The basic procedure is to systematically lay a series of grids of decreasing calibre (the boxes) over an image and record data (the counting) for each successive calibre (Mandelbrot, 1983; Smith *et al.*, 1996; Khokha, 1996; Liu *et al.*, 2003). Fractal dimension is then calculated using an equation (1). For grayscale images, the average value of pixels per box is used as a size of a set. The images have been processed using FracLac

plugin to ImageJ (open source freeware), dedicated to fractal analysis of digital images. One of the options of the software – *SubScan*, allows to calculate a local fractal dimension. The image is divided into fragments, so-called samples – for each sample a fractal dimension is calculated independently.

The methodology of the extraction of built-up area mask using fractal analysis was as follow (also look fig.3):

1. Converting images to greyscale.
2. Fractal analysis. Applying box counting method using *SubScan* application. Different sample sizes have been tested (more details about it will be presented in the section 4.1)
3. Thresholding. Built-up areas generally are marked with higher fractal dimension, than other land use classes so binarization with the upper threshold has been applied to extract built up area masks. Different values of threshold have been tested (more details about it will be presented in the section 4.1).
4. Accuracy assessment. Comparison of the built up area masks extracted using fractal analysis to the reference masks obtained using visual interpretation (fig. 2).

Granulometric maps. Granulometric maps are the concept derived from mathematical morphology granulometry. Firstly, mathematical morphology itself, and then, morphological granulometry and granulometric maps will be briefly presented.

Mathematical morphology is a set theory approach, developed by J.Serra and G. Matheron (Serra, 1982, 1986, 1988; Haralick *et al.*, 1987) It provides an approach to processing digital images based on a contextual information, like geometric shape, texture, neighborhood etc., depending on a type of morphological process applied (Nieniewski, 1998, 2005; Kupidura, 2006; Kupidura *et al.*, 2010). It bases on two fundamental operations called erosion and dilation, defined by the following formulas:

$$\varepsilon_B(f) = \inf \{g(f - y), y \in B\} \quad (2)$$

where: $\varepsilon_B(f)$ is an erosion of a function (image) f using a structuring element (SE) B .

Analogically, dilation may be defined as follows:

$$\delta_B(f) = \sup \{g(f + y), y \in B\} \quad (3)$$

where: $\delta_B(f)$ is a dilation of a function (image) f using a structuring element B .

Two other operations are not literally elementary (they are combinations of erosion and dilation), but very often they are called and treated this way, because they constitute parts of a majority of morphological operations, more often, than erosion or dilation alone. First of them, opening is defined by the following equation:

$$\gamma_B(f) = \delta_B(\varepsilon_B(f)) \quad (4)$$

and a second one, closing, as:

$$\varphi_B(f) = \varepsilon_B(\delta_B(f)) \quad (5)$$

Morphological granulometry is a transformation allowing to compute the size distribution of objects in an image. It produces a set of images, showing the results of a subtractions between the images sequently opened or closed (morphologically) using a structuring elements of ascending size. Propositions of application of this operation for a remote sensing data processing appeared before e.g. in (Flouzat, 1989; Kupidura, 2010; Kupidura *et al.*, 2010).

Theoretical fundamentals of granulometry are presented below in the following equations. Let's consider such a set of structuring elements B_n with $n = 1, 2, 3, \dots, N$, describing a size of a structuring element, that $B_n = nB$. Let's consider as an initial image, and f_n , for – a sequence of openings (4) using a successive structuring elements:

$$f_1 = \gamma_{B_1}(f), f_2 = \gamma_{B_2}(f_1), \dots, f_n = \gamma_{B_n}(f_{n-1}) \quad (6)$$

Opening is an anti-extensive operation, so if $S(f_n)$ stands for a cardinality of an image (the sum of pixel values) (Nieniewski, 1998; Kupidura 2006) it is true, that:

$$s(f) \geq s(f_1) \geq s(f_2) \geq s(f_3) \dots \geq s(f_n) \quad (7)$$

Now, basing on the image numbers (7), we can calculate a size distribution (Dougherty *et al.*, 1992)

$$SD_n = \frac{s(f) - s(f_n)}{s(f)} \quad (8)$$

Size distribution (8) may be used to calculate granulometric density:

$$dSD_n = SD_{n+1} - SD_n \quad (9)$$

A set of granulometric density values for different n values is called a granulometric function.

It is possible to use a closing operation instead of opening. This kind of operation is called anti-granulometry.

The main idea of the granulometric maps is to produce the images containing information about a local granularity. This is achieved by calculating a set of granulometric density values only for a neighbourhood of each pixel, independently (Kupidura, 2015). This way, from a one source image a set of images (granulometric maps) is produced, where pixel values might be interpreted as an amount of objects of specific size in the neighbourhood of the pixel (granulometry window). We can also say, that for every pixel, a discreet function, describing its granularity is produced and written as a set of granulometric maps.

The methodology of the extraction of built-up area mask using granulometric maps was as follow (also look fig. 3):

1. Converting images to grayscale.
2. Granulometry. Creation of a set of granulometric maps using BlueNote – a freeware dedicated to mor-

phological image processing. The granulometric maps produced using both, opening (granulometry) and closing (anti-granulometry) approaches have been stacked into a one multi-layer image. Different sizes of granulometry windows have been tested (more details about it will be presented in the section 4.2).

3. ISODATA classification. It has been performed with an initial class number – 50. After that, the image has been reclassified to the binary mask of built up area (and non-built up area).
4. Accuracy assessment. Comparison of the built up area masks extracted using fractal analysis to the reference masks obtained using visual interpretation (fig. 2).

Results

Fractal analysis. In the part of the research concerning fractal analysis, more than 40 different sample sizes have been tested. This preliminary step showed, that the best results might be obtained using samples of a size 40 pixels.

The next step was about to find the value of fractal dimension threshold, allowing the extraction of built-up area the most efficiently. All fractal dimension values have been checked with 0,001 precision, in the range from 2,500 to 2,560. To evaluate the results, kappa coefficient of agreement (Landis and Koch, 1981) has been used. Also, additional coefficients have been calculated, to improve the evaluation: commission and omission errors (for both final classes – built-up area and non built-up area) and overall accuracy.

The best results of built-up area extraction using fractal analysis are presented in the table 1. The final masks of a built-up area are presented in the figure 4.

As the table 1 shows, that for different test areas, the best values are slightly different (2,532 for Jablonna and 2,530 for Karczew), but we may affirm, that the overall best results are obtained using 2,530 as a value of fractal dimension, the more so, because the difference of the accuracy (assessed using Kappa coefficient) for Jablonna test area, using 2,530 or 2,532 as the values of the threshold is negligible (0,512 vs. 0,513). The stability of the most efficient parameters indicates, that there is a possibility to relate them to the spatial resolution of the image (0,65m x 0,65m in the presented case) and a sample size in order to obtain an optimal accuracy of the classification.

The results themselves, however, are moderately satisfying. According to Landis and Koch (1977) values close to 0,5 (from 0,4 to 0,6) indicate a moderate agreement of two data sets: one extracted using a fractal analysis and a referential one, based on a photo interpretation; other literature references (Fleiss, 1981) interpret values from 0,4 to 0,75 as “from fair to good”.

A noticeable difference between the results for different test areas probably is caused by the relatively significant area of wooded land in the south west part

Table 1. Results of the fractal analysis approach. Sample size 40 pixels.

Tabela 1. Wyniki podejścia opartego na analizie fraktalnej. Rozmiar próbkowania 40 pikseli.

Fractal dimension threshold <i>Próg wymiaru fraktalnego</i>	Commission error – built-up area [%] <i>Błąd nadmiaru – tereny zabudowane [%]</i>	Commission error – non built-up area [%] <i>Błąd nadmiaru – tereny niezabudowane [%]</i>	Omission error – built-up area [%] <i>Błąd pominięcia – tereny zabudowane [%]</i>	Omission error – non built-up area [%] <i>Błąd pominięcia – tereny niezabudowane [%]</i>	Overall accuracy [%] <i>Ogólna dokładność [%]</i>	kappa coefficient <i>współczynnik kappa</i>
Jabłonna						
2,529	65,9	86,4	38,6	11,4	55,5	0,510
2,530	65,3	86,7	37,7	11,9	55,4	0,512
2,531	64,7	86,9	36,7	12,4	55,4	0,512
2,532	64,1	87,2	35,6	13,0	55,3	0,513
2,533	63,4	87,4	34,7	13,6	55,1	0,512
Karczew						
2,529	72,0	87,8	36,6	8,6	84,2	0,570
2,530	70,7	88,2	34,7	9,4	84,1	0,573
2,531	69,1	87,8	36,1	9,9	83,3	0,553
2,532	68,4	88,0	35,1	10,4	83,2	0,554
2,533	67,8	88,3	34,0	10,9	83,1	0,556

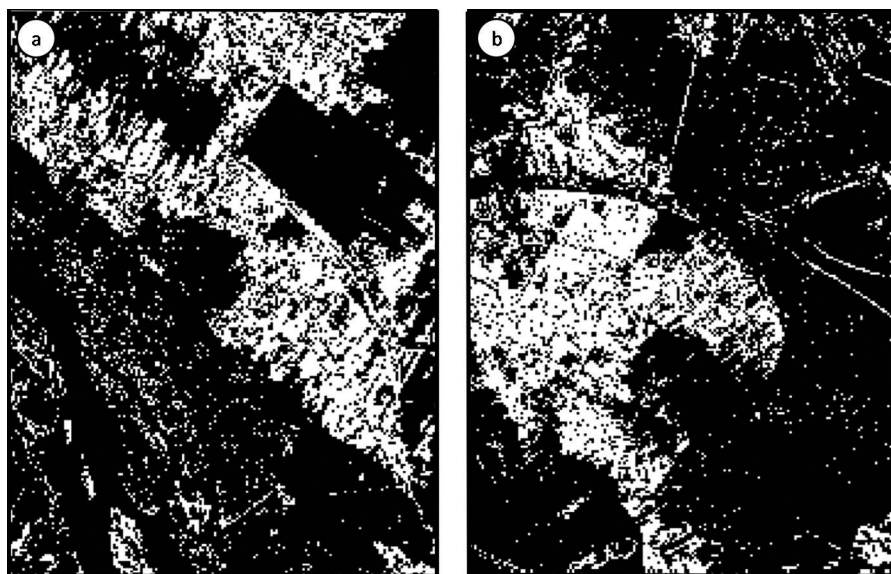


Fig. 4. Results of the fractal analysis approach; built-up area marked with white color, a) Jabłonna, b) Karczew.

Ryc. 4. Wyniki podejścia opartego na analizie fraktalnej; tereny zabudowane zaznaczone białym kolorem, a) Jabłonna, b) Karczew.

of Jabłonna test area. The granularity of the image of this class of land cover is relatively high, so in some parts, fractal dimension values might be as high, as the values of built-up area, and this is a probable cause of a lesser accuracy of the Jabłonna test area mask.

Granulometric maps. In this part of the research, several granulometric window sizes have been tested. Afterwards, the granulometric maps produced using different parameters have been classified using ISO-DATA algorithm to extract (finally, after reclassification process) a binary mask of built-up area. The results are presented in the table 2. The same measures of accuracy of the classification, as for the evaluation of the fractal analysis approach, have been used: kappa coefficient of agreement as the main measure and, additionally, com-

mission and omission errors for both classes, and overall accuracy. The final masks are presented in the figure 5.

For both test areas, granulometric window of the radius of 25 pixels size have been proof the best. The results of the granulometric approach are significantly better than those, obtained using a fractal analysis approach. Interpreting the values of kappa coefficient we may describe the agreement of the extracted mask with the reference data as substantial (according to an interpretation of kappa coefficient proposed by Landis and Koch (1977)), or, for Jabłonna and Karczew, respectively, as good and excellent, according to Fleis (1981).

Once again, the accuracy of the results of Karczew test area is noticeably better. The reason is similar –

Table 2. Results of the granulometric approach. R20, R25 and R30 mean radius (in pixels) of granulometric window – a neighborhood of pixel taken into account for granulometry calculations

Tabela 2. Wyniki podejścia opartego na analizie granulometrycznej. R20, R25 i R30 oznaczają promień (w pikselach) okna granulometrii – sąsiedztwa branego pod uwagę w obliczeniach granulometrycznych

Granul. window radius <i>Promień okna granulometrii</i>	Commission error – built-up area [%] <i>Błąd nadmiaru – tereny zabudowane [%]</i>	Commission error – non built-up area [%] <i>Błąd nadmiaru – tereny niezabudowane [%]</i>	Omission error – built-up area [%] <i>Błąd pominięcia – tereny zabudowane [%]</i>	Omission error – non built-up area [%] <i>Błąd pominięcia – tereny niezabudowane [%]</i>	Overall accuracy [%] <i>Ogólna dokładność [%]</i>	kappa coefficient <i>współczynnik kapp</i>
Jabłonna						
R20	73,4	93,1	18,4	10,6	87,3	0,685
R25	74,5	92,6	19,9	9,9	87,5	0,686
R30	73,2	92,9	18,9	10,7	87,2	0,680
Karczew						
R20	77,3	94,7	14,6	8,7	89,8	0,742
R25	78,1	95,1	13,5	8,4	90,3	0,754
R30	74,3	95,8	11,1	10,7	89,2	0,735

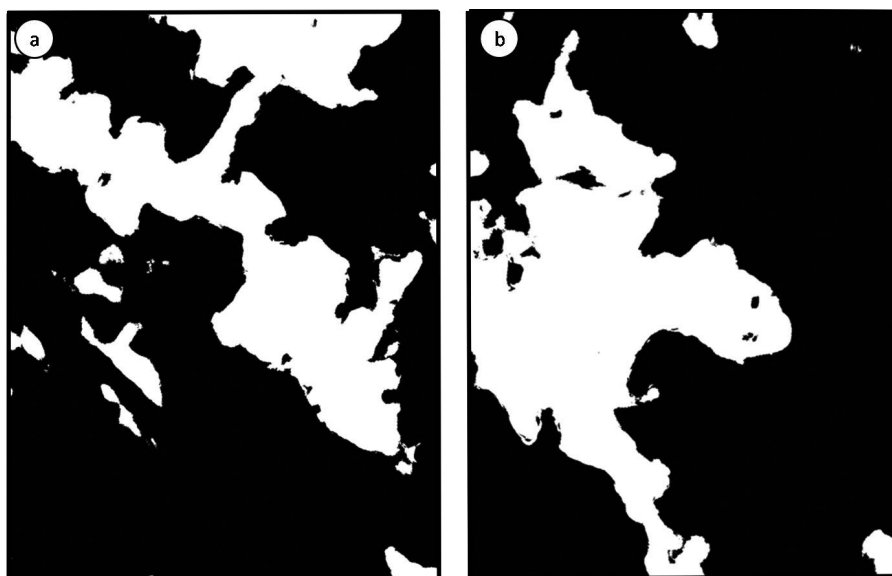


Fig. 5. Results of the granulometric approach; built-up area marked with white color, a) Jabłonna, b) Karczew.

Ryc. 5. Wyniki podejścia opartego na analizie granulometrycznej; tereny zabudowane zaznaczone białym kolorem, a) Jabłonna, b) Karczew.

some parts of the wooded area in the south-west fragment of Jabłonna test image have been misclassified as built-up area (fig. 5). However, the results for the both test areas may be considered as satisfying.

Conclusions

The research showed the superiority of the granulometric approach over the presented fractal analysis approach. The results of a classification basing on the granulometric maps are significantly better than the reclassification basing on a fractal dimension value. However, it should be mentioned, that the better results of a granulometric analysis might be related to the extensive manner of the interpretation on built-up area. Besides, the fractal analysis approach might be still improved. The algorithm implemented in the existing software (like FracLac used for the research) bases on

a fixed grid of samples what limits a spatial resolution of the output image to the size of the sample. What might be considered is to found the general idea of a box counting method on a granulometric maps scheme – to calculate a fractal dimension for each and every pixel (or rather for its neighbour) independently. The improvement of a spatial resolution might also improve the result of a fractal analysis from the point of view of image classification.

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